

# Limit distributions for sums of independent

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1954

[1]B. V. Gnedenko and A. N. Kolmogorov: Limit distributions for sums of independent random variables Addison-Wesley Mathematics Series. Addison-Wesley (Cambridge, MA), 1954. Translated and annotated by K. L. Chung. With an Appendix by J. L. Doob.[MR0062975](#) [Zbl 0056.36001](#) book

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AUTHOR = {Gnedenko, B. V. and Kolmogorov, A. N.},
TITLE = {Limit distributions for sums of independent
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SERIES = {Addison-Wesley Mathematics Series},
PUBLISHER = {Addison-Wesley},
ADDRESS = {Cambridge, MA},
YEAR = {1954},
PAGES = {ix+264},
NOTE = {Translated and annotated by K. L. Chung.
With an Appendix by J. L. Doob. MR:0062975.
Zbl:0056.36001.},
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1968

[2]B. V. Gnedenko and A. N. Kolmogorov: Limit distributions for sums of independent random variables, revised edition. Addison-Wesley, 1968. Translated from the Russian, annotated, and revised by K. L. Chung. With appendices by J. L. Doob and P. L. Hsu.[MR0233400](#) book

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by J.~L. Doob and P.~L. Hsu. MR:0233400.},
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1981

[3]Seminar on stochastic processes, 1981 (Northwestern University, Evanston, IL, April 1981). Edited by E. Çinlar, K. L. Chung, and R. K. Getoor. Progress in Probability and Statistics 1. Birkhäuser (Boston, MA), 1981. [MR 647778](#) [Zbl 0471.00025](#) book

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1983

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ISBN = {0387907254, 9780387907253},

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1984

[6]Seminar on stochastic processes, 1983 (University of Florida, Gainesville, FL, 1983). Edited by E. Çinlar, K. L. Chung, and R. K. Gettoor. Progress in Probability and Statistics 7. Birkhäuser (Boston, MA), 1984. [MR 902407](#) [Zbl 0546.00025](#) book

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1987

[9]Seminar on stochastic processes, 1986 (University of Virginia, Charlottesville, VA, March, 1986). Edited by E. Çinlar, K. L. Chung, R. K. Getoor, and J. Glover. Progress in Probability and Statistics 13. Birkhäuser (Boston, MA), 1987. [MR 902421](#) [Zbl 0607.00014](#) book

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1988

[10]Seminar on stochastic processes, 1987 (Princeton University, Princeton, NJ, March 26–28, 1987). Edited by E. Çinlar, K. L. Chung, R. K. Getoor, and J. Glover. Progress in Probability and Statistics 15. Birkhäuser (Boston, MA), 1988. [MR 1046408](#) [Zbl 0635.00011](#) book

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1989

[11]Seminar on stochastic processes, 1988 (University of Florida, Gainesville, FL, March 3–5, 1988). Edited by E. Çinlar, K. L. Chung, R. K. Getoor, and J. Glover. Progress in Probability 17. Birkhäuser (Boston, MA), 1989. [MR 990471](#) [Zbl 0659.00010](#) book

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[12]Seminar on stochastic processes, 1989 (University of California, San Diego, CA, March 30–April 1, 1989). Edited by E. Çinlar, K. L. Chung, R. K. Getoor, P. J. Fitzsimmons, and R. J. Williams. Progress in Probability 18. Birkhäuser (Boston, MA), 1990. [MR 1042337](#) [Zbl 0686.00017](#) book

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1992

[13]Seminar on stochastic processes, 1991 (University of California, Los Angeles, CA, March 23–25, 1991). Edited by E. Çinlar, K. L. Chung, M. J. Sharpe, P. Fitzsimmons, S. Port, and T. Liggett. Progress in Probability 29. Birkhäuser (Boston, MA), 1992. Dedicated to the memory of Steven Orey. [MR 1172137](#) [Zbl 0745.00038](#) book

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YEAR = {1992},
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ISSN = {0892-063X},
ISBN = {0817636285, 9780817636289},
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[14]Seminar on stochastic processes, 1992 (University of Washington, Seattle, WA, March 26–28, 1992). Edited by E. Çinlar, K. L. Chung, M. J. Sharpe, R. F. Bass, and K. Burdzy. Progress in Probability 33. Birkhäuser (Boston, MA), 1993. [MR 1278074](#) [Zbl 0780.00020](#) book

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ADDRESS = {Boston, MA},
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Well, we know that one of our goals for this lesson is to find the probability distribution of the sample mean when a random sample is taken from a population whose measurements are normally distributed. Then, let's just get right to the punch line! Well, first we'll work on the probability distribution of a linear combination of independent normal random variables  $(X_1, X_2, \dots, X_n)$ . On the next page, we'll tackle the sample mean! Theorem. If  $(X_1, X_2, \dots, X_n)$  are mutually independent normal random variables with means  $(\mu_1, \mu_2, \dots, \mu_n)$  and variances  $(\sigma_1^2, \sigma_2^2, \dots, \sigma_n^2)$ , then the distribution of  $\sum_{i=1}^n a_i X_i$  is normal with mean  $\sum_{i=1}^n a_i \mu_i$  and variance  $\sum_{i=1}^n a_i^2 \sigma_i^2$ . In mathematics, specifically in the theory of generalized functions, the limit of a sequence of distributions is the distribution that sequence approaches. The distance, suitably quantified, to the limiting distribution can be made arbitrarily small by selecting a distribution sufficiently far along the sequence. This notion generalizes a limit of functions; a limit as a distribution may exist when a limit of functions does not. The distribution of a sum of independent random variables. The general case, the discrete case, the continuous case. Examples. Exercises. Suppose  $X_1, X_2, \dots, X_n$  are mutually independent random variables and let  $S_n$  be their sum: The distribution of  $S_n$  can be derived recursively, using the results for sums of two random variables given above: first, define  $f_1(x)$  and compute the distribution of  $X_1$ ; then, define  $f_2(x)$  and compute the distribution of  $X_1 + X_2$ ; and so on. Central Limit Theorem. Moment generating function. Combinations.

is a chi-squared distribution with  $k-1$  degrees of freedom. What happens in the limit? What is the distribution of  $Y = \int_0^1 Z_t^2 dt$ ? Thanks in advance. To see this, note that, on every interval of the subdivision, infinitely many independent random variables  $(U_t)$  are involved, whose distributions have common support  $S=[0,+\infty)$ , hence their supremum is the supremum of  $S$  and their infimum is the infimum of  $S$ . As a consequence, almost surely the function  $t \mapsto U_t$  is not Riemann(-Darboux) integrable and  $Y$  does not exist. (That is, unless one precises another way to define the integral.) Edit (Added to answer a question raised by @Sasha.)

87 part ii. general limit theorems chapter 4. General Limit Theorems for Sums of Independent Summands 94 § 20. Statement of the problem. Sums of infinitely divisible summands . Unimodality of distributions of the class L 157 part iii. identically distributed summands chapter 7. Fundamental Limit Theorems 162 § 33. Statement of the problem. Stable laws 162 § 34. Darling, Donald A. Review: B. V. Gnedenko and A. N. Kolmogorov, Limit distributions for sums of independent random variables. Bull. Amer. A Local Limit Theorem for Large Deviations of Sums of Independent, Nonidentically Distributed Random Variables McDonald, David, Annals of Probability, 1979. Infinite  $F$ -Divisibility of Integer-Valued Random Variables James, Ian R., Annals of Probability, 1976. The Asymptotic Distribution of Intermediate Sums Csorgo, Sandor and Mason, David M., Annals of Probability, 1994. A Central Limit Theorem for Sums of Interchangeable Random Variables Chernoff, H. and Teicher, H., Annals of Mathematical Statistics, 1958. euclid.bams/1183520449. Browse.